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Highlights

- We present a solution method for the liner shipping network design problem.
- The algorithm explicitly handles transshipment time limits for all demands.
- Individual sailing speeds at each service leg are used to balance sailing speed against operational costs.
- Computational results are showing very promising results for realistic global liner shipping networks.
- A sensitivity analysis on fluctuations in bunker price confirms the applicability of the algorithm.

Competitive Liner Shipping Network Design

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Abstract

We present a solution method for the liner shipping network design problem which is a core strategic planning problem faced by container carriers. We propose the first practical algorithm which explicitly handles transshipment time limits for all demands. Individual sailing speeds at each service leg are used to balance sailing speed against operational costs, hence ensuring that the found network is competitive on both transit time and cost. We present a matheuristic for the problem where a MIP is used to select which ports should be inserted or removed on a route. Computational results are presented showing very promising results for realistic global liner shipping networks. Due to a number of algorithmic enhancements, the obtained solutions can be found within the same time frame as used by previous algorithms not handling time constraints. Furthermore, we present a sensitivity analysis on fluctuations in bunker price which confirms the applicability of the algorithm.

Keywords: container shipping, network design, level of service

1. Introduction

Given a fleet of container vessels and a selection of ports, the classical *Liner Shipping Network Design Problem (LSNDP)* constructs a set of scheduled routes (*services*) with a fixed frequency for container vessels to provide transport for containers worldwide (Brouer et al. [4]). This paper presents the *Competitive Liner Shipping Network Design Problem (CLSNDP)* extending the classical LSNDP to consider *level of service*, i.e. the transit time provided for a given cargo as well as the transportation cost charged. These two parameters are the main concern for customers, and hence they are crucial parameters for designing competitive networks.

The classical LSNDP is offset in the main objective of the carrier; to maximize profit through the revenues gained from container transport taking into account the fixed cost of deploying vessels and the variable cost related to the operation of the services. The opposing objectives of the customer and the carrier represents an inherent trade-off in the design of a liner shipping network. Minimizing the cost of the network will provide low freight rates, but are likely to result in prolonged transit times as shown by Karsten et al. [15]. On the other hand, designing a network to minimize transit times is likely to result in a very costly network favoring direct connections at high sailing speeds.

The models for the classical LSNDP differ on two traits. First, the ability to model and charge transshipments between services. Containers are often not transported directly from their port of origin to their port of destination, and hence it is important to be able to handle the time and cost of transshipments. Second, models differ on requiring a fixed frequency of service or providing flexibility in the frequency. A service is cyclic but may be non-simple, that is, ports can be visited

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more than once. In this model we allow a single port to be visited twice, yielding a so-called *butterfly* route.

The paper by Agarwal and Ergun [1] imposes a weekly frequency of service and allows for transshipment, but the model cannot cater for the handling cost associated with transshipments. The paper by Alvarez [2] can cater for transshipment and transshipment costs (except within butterfly services) and allows for flexible frequencies of service. In Reinhardt and Pisinger [24] each vessel is treated separately allowing flexible frequencies, and the model allows for transshipment costs also on butterfly routes. Brouer et al. [4] provides an analysis of the real life requirements and present a reference model for the classical LSNDP. The model is offset in Alvarez [2] accounting correctly for transshipments on all services and allowing both flexible and fixed frequencies. The above models are all variants of specialized capacitated network design problems.

Meng et al. [19], Christiansen and Fagerholt [8], Christiansen et al. [9] provide broader reviews of recent research on routing and scheduling problems within liner shipping. In the literature several papers extend the classical LSNDP e.g. by incorporating intermodal considerations (Liu et al. [17]) or aiming to narrow the definition of service (Plum et al. [21]). However, it is generally acknowledged that considering level of service is the most important extension to the classical LSNDP because it is the decisive factor in designing a competitive network (Alvarez [3], Brouer et al. [4]). Two approaches for considering level of service has been suggested in the literature. The first method is to include inventory cost in a multi-criteria objective function as seen in Alvarez [3]. Inventory cost is primarily a concern to the shipper and the idea of introducing it for the carrier is to ensure that longer transit times will result in lower freight rates. However, the bilinear expression proposed by Alvarez [3] is not computationally tractable. Another approach is to impose restrictions on the allowed transit times for each container. The idea here is that the carrier needs to provide competitive transit times in a market of several players. Wang and Meng [30] introduce deadlines on cargo in a non-linear, non-convex mixed-integer programming (MIP) formulation of a LSNDP. A drawback of this formulation is that it cannot cater for transshipments of cargo which is the backbone of global liner shipping networks. Recently Karsten et al. [14] presented a capacitated multi-commodity network design formulation that imposes transit time restrictions while still allowing transshipments between services and Karsten et al. [15] showed that time restricted multi-commodity flow problem arising as a sub-problem can be efficiently solved for a large global shipping network. The CLSNDP in this paper builds upon these contributions.

Introducing transit time restrictions is essential in the LSNDP from a customer perspective, but to maintain low fuel (bunker) cost this must be accompanied by modelling the services with variable speed. Traditionally, models of the LSNDP operate with a constant speed on services although variable speed on each leg is used in practice. In a network with constant speed the most transit time restricted commodity will force the entire service to speed up, and hence increase the bunker consumption of the service unnecessarily with a resulting increase in both cost and CO_2 emissions. Figure 1 illustrates the problem of maintaining constant speed during the design process. The container entering at A and leaving at B, k_{AB}, has the tightest transit time requirement among the containers currently transported on service s with a transit time restriction of 3 days, which requires a speed of 14 knots. This results in a deployment of 2 vessels at a speed of nearly 21 knots, because of only two possible deployments with constant speed and the weekly frequency requirement imposed. If speed can be determined individually on each sailing leg, 3 vessels can be deployed with a speed of 14 knots between A and B and a speed of 12 knots on the remaining sailing legs maintaining the weekly frequency but resulting in a significant decrease in the bunker consumption (since the bunker usage per unit of distance is a quadratic function of speed (Brouer et al. [4])). The decrease in bunker cost must be evaluated against the cost of deploying an additional vessel on the service. The computational results presented in Karsten et al. [14] support a higher average speed and low fleet deployment in networks optimized with transit time restrictions

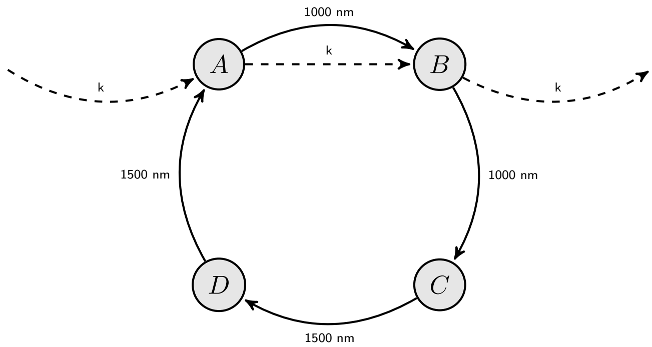

Figure 1: A service s illustrated with constant speed and weekly frequency. The nodes are ports and the solid lines correspond to sailing arcs. Two deployments are possible to complete the round trip of 5,000 nm (nautical miles) within the speed bounds: Three vessels deployed ($D^{e(s)} = 3$) results in a constant speed of 12.3 knots, while two vessels deployed ($D^{e(s)} = 2$) results in a constant speed of 20.8 knots. The most transit time critical commodity, k, on the service is for the commodity illustrated by the dashed line from A to B, where the transit time restriction is 3 days requiring a speed of 14 knots.

and constant speed.

Therefore, the CLSNDP is extending the reference model for LSNDP (Brouer et al. [4]) to consider transit time restrictions coupled with variable speed on each sailing leg in order to properly address the trade-off between providing competitive transit times, while reducing cost as well as CO_2 emissions. In this paper we propose the first algorithm to solve CLSNDP by an adaptation of the matheuristic of Brouer et al. [6] that considers transshipment times and optimize speed on each sailing leg. The underlying basis for the model is a capacitated multi-commodity network design formulation where we can accurately model transshipment operations, cost structures, and restrictions on container transit time of individual containers. The formulation adheres to the objective and constraints of Brouer et al. [4] with a fixed weekly frequency. As we are not solving the mathematical formulation using an exact algorithm we have chosen to place the mathematical model in Appendix A.

Speed optimization in maritime transportation has received quite a lot of interest in the literature across economics and operations research over the past decade. Psaraftis and Kontovas [23] survey models and taxonomy on speed optimization and in Psaraftis [22] “slow steaming” as a phenomenon is discussed. Notteboom and Vernimmen [20] and Ronen [26] provide insights on speed optimization in liner shipping and show the importance of optimizing speed in liner shipping networks by studying a single service. There are numerous examples of speed optimization within liner shipping e.g. the non-linear MIP formulation presented in Wang and Meng [29], or speed optimization coupled with fleet deployment in e.g. Gelareh and Meng [11], Meng and Wang [18], Zacharioudakis et al. [31]. A number of contributions are concerned with the coupling between transit time and speed in optimizing the network (Cheaitou and Cariou [7], Wang and Meng [27, 28]). Reinhardt et al. [25] present a MIP model for adjusting the port berth times such that the bunker consumption is minimized while retaining the customer transit times. A penalty is assigned to each change of berth time in order to limit the number of changes. Karsten et al. [16] use Benders decomposition to simultaneously optimize sailing speed and container routing. All containers have an associated limit on the transit times that needs to be met.

Deciding an optimal speed configuration in a liner shipping network requires consideration of the network in its entirety as transit times of commodities may be decided by several interoperating services. Likewise commodity paths are likely to change with the speed optimization if cargo routings are flexible. However, computational results from the above mentioned papers indicate that this is not computationally tractable for revaluation in a large-scale heuristic search. The matheuristic for the CLSNDP proposed in this paper is considering speed as one of the dimensions in the solution space and therefore a fast method for optimizing speed is needed. In tramp shipping speed optimization of an isolated route in the network is optimal. Variable speed for a single ship route in tramp shipping has been explored in Fagerholt et al. [10], I. Norstad and Laporte [13], Hvattum et al. [12], where the introduction of speed optimization allowing variable speed on a sail route results in significant fuel savings. In Fagerholt et al. [10] a MIP with a non-linear objective function depicting the vessels bunker consumption as a function of speed is presented. The speed optimization problem can be transformed into a directed acyclic graph if speeds are discretized and the resulting speed profile is simply a shortest path, which can be efficiently calculated for a directed acyclic graph. The approach by Fagerholt et al. [10] cannot be adopted directly, since a liner shipping service will be carrying multiple commodities and hence the time windows are defined per pickup node. Transforming the problem into a graph would result in node specific time windows accounting for times between every OD pair assigned to the service, which would require a resource constrained shortest path with a specific resource for every port in the service. This is unlikely to be efficiently solved. However, we can adapt the non-linear MIP formulation of Fagerholt et al. [10] to optimize speed on a single service given constraints on the slack time of each commodity currently transported on the service. We also consider opportunity cargo not currently transported, as speed optimization may lead to new attractive transport opportunities. The non-linear bunker consumption function is approximated by a piecewise linear function of the time to sail a given leg and the speed optimization MIP can be efficiently solved using a standard MIP solver making it suitable to incorporate into a heuristic. Our computational results show that it is tractable to incorporate level of service in the network design process by considering container transit time restrictions *and* variable speed in a heuristic context, and we are able to design profitable networks for scenarios resembling global liner shipping networks.

The rest of the article is organized as follows. Section 2 discusses the extensions from the LSNDP to the CLSNDP. Section 3 gives an overview of our solution method and describe the level of service implications in detail. Section 4 presents computational results on realistic instances from the benchmark suite *LINER-LIB* before we conclude and discuss future work in Section 5.

2. Problem description

Given a fleet of container vessels and a selection of ports, the CLSNDP constructs a set of services to provide transport for containers worldwide. It extends the classical LSNDP to consider level of service as this is the main concern for the shipper. The CLSNDP we present here is based on the reference model for the LSNDP presented in Brouer et al. [4] which has been extended in Karsten et al. [14] to consider transit time restrictions for all commodities.

An instance of the CLSNDP consists of the set of ports, P, with an associated port call cost c_p^e for vessels of class $e \in E$, (un)load cost c_p^U, c_p^L, transshipment cost c_p^T and berthing time B_p spent in port p. Furthermore, we have a set of demands, K, available for transport each week where each demand has an origin $o_k \in P$, a destination $d_k \in P$, a quantity, q_k, a revenue per unit, z_k, a reject penalty per unit $\tilde{z}_k$ and a maximal transit time, $\bar{t}_k$. To service the routes, there is a set of vessel classes, E, with specifications for the weekly charter rate, C^e, capacity U^e, minimum (v^e_{min}) and maximum (v^e_{max}) speed limits in knots per hour, and bunker consumption per hour, when the vessel is idle at ports h^e. The bunker consumption is a function $g^e(t, d)$ of the sailing time t, distance d and vessel class e. There are Ψ^e vessels available of class $e \in E$. The price for one metric ton of bunker is denoted β. Finally we have a matrix of the direct distances d^e_{ij} between all pairs of ports $i, j \in P$ and for all vessel classes $e \in E$. The distance may depend on the vessel class draft as the Panama Canal is draft restricted. Along with d^e_{ij} follows an indication of the cost l^e_{ij} associated with a possible traversal of a canal.

A solution to the CLSNDP is a (small) subset S of the set of all feasible services $\mathcal{S}$. A feasible service s consists of a route, a number of vessels, and a vector of sailing speeds corresponding to each sailing leg such that the total round trip time is a multiple of a week. A route is an ordered set of ports $P' \subseteq P$. Several services may in principle use the same route, although it generally is cheaper to aggregate the two services into one service operated by a larger vessel class.

A weekly frequency of port calls is obtained by deploying multiple vessels to a service. Let $e(s) \in E$ be the vessel class assigned to a service s and $D^{e(s)}$ the number of vessels of class $e(s)$ required to maintain a weekly frequency. A round trip may last several weeks but due to the weekly frequency exactly one round trip is performed every week. The service time T_s is the time needed to complete the cyclic route.

A full model of the problem is found in Appendix A. The primary change in order to accommodate transit time restrictions into the model of Brouer et al. [4] is to decompose the multi commodity flow problem into a path flow formulation. In the path flow formulation only paths respecting the maximal transit time for a given commodity are feasible. This extension of the LSNDP with transit time restrictions is a non-compact formulation with integer service variables defining a port call sequence, a vessel type, number of ships and a constant speed, and real path variables for routing the commodities. As transit times are closely linked to speed, the constant speed needed to accommodate transit time restrictions will generally be determined by the commodity with the most restrictive transit time. However, it is unnecessary to maintain a high speed throughout the service if this commodity is only carried on part of the service. Therefore we use service variables that include variable speed by allowing each sailing to take on any speed within the feasible speed interval, while maintaining a weekly frequency of service. The overall objective of CLSNDP is to maximize profit, however, the extensions potentially results in fuel savings and/or a larger cargo uptake in the network along with ensuring a competitive level of service in the network.

The next section provides a broad overview of the algorithm and its components. The overview includes the extensions necessary to enable consideration of level of service, namely transit time restrictions for each individual commodity and optimizing speed on each sailing in the network. Following the overview the extensions will be described in further detail.

3. Algorithm

The proposed matheuristic is based on the algorithm from Brouer et al. [6]. In every iteration the solution x consists of a set S of services, each service $s \in S$ having an associated vessel class $e(s)$ and sailing time $t_{j,j+1}$ between ports j and $j+1$. The sailing time implicitly gives the speed between a pair of ports depending on the distance and for each pair of ports on a service the speed is allowed to be different.

Since the evaluation of the objective function makes it necessary to flow all containers through the network, only a limited number of iterations can be evaluated throughout the search, and therefore it is important to use a large neighborhood search, combined with a shrewd way of choosing the direction of the search.

Algorithm 1 presents high level pseudocode for the overall matheuristic. Initially a solution is constructed by dividing the available fleet onto services. Subsequently the services are populated with port calls following a greedy parallel insertion procedure according to the distance and the trade volume between ports in the service in line 1. The subsequent search for improved solutions is guided by a simulated annealing scheme in the while loop of lines 5–25. The cooling scheme consists of the initial temperature defined as the average of the sum of the absolute difference between the first ten solutions found. The temperature is cooled by a constant ratio $\alpha = 0.98$ and the stopping citerion is the $MINtemp = 0.01$. The simuated annealing will accept a new solution x' if $exp((z(x) - z(x'))/temp) > random[0, 1)$ in line 12. The primary component of the matheuristic

Algorithm 1 High Level algorithm for CLSNDP

Require: An instance of the CLSNDP
1: Construct an initial solution x using a greedy algorithm
2: Set the best known solution $x^* = x$
3: Set the iteration counter $iter = 0$
4: Set the initial temperature $temp = temp_0$
5: **while** $temp > MINtemp$ **and** $time < MAXtime$ **do**
6:   **for** each service $s \in x$ **do**
7:     $x' \leftarrow x \setminus \{s\}$
8:     $s' \leftarrow IP(s)$: improve solution by insertion/removal of port calls on service s
9:     Resolve cargo flow
10:     Optimize speed of each sailing on s'
11:     $x' \leftarrow x' \bigcup \{s'\}$
12:     **if** accept solution **then**
13:       Set $x \leftarrow x'$
14:       Possibly update best known solution: $x^* \leftarrow x$
15:   $iter \leftarrow iter + 1$
16:   $temp \leftarrow temp \cdot \alpha$
17:   **if** $iter \mod 4 = 0$ **then**
18:     Apply reinsertion heuristic to obtain new solution x' with promising butterfly routes
19:     **if** solution improves **then**
20:       Set $x \leftarrow x'$
21:       Possibly update best known solution: $x^* \leftarrow x$
22:   **if** $iter \mod 10 = 0$ **then**
23:     Apply perturbation to obtain a solution x' with a different service composition
24:     Set $x \leftarrow x'$
25:     Possibly update best known solution: $x^* \leftarrow x$
26: **return** x^*

is a neighbourhood for inserting and removing port calls on a single service which is formulated as an integer program in line 8. The integer program is described in detail in Section 3.1. In order to optimize speed in the network a heuristic method based on a non-linear MIP is applied. The heuristic optimizes the speed of all legs on a single service given the time limits of cargo currently transported on this service and the time limit of opportunity demands, that are currently rejected due to transit time restrictions. This MIP is called in line 10 after resolving the multicommodity flow problem in line 9 given the changes to service s. As changes are only made to a single service, the column generation algorithm used is warm started using the technique described in Brouer et al. [6]. The simulated annealing scheme decides whether the new solution is accepted. The reinsertion heuristic in line 18 introduces butterfly ports on promising candidate services. The perturbation heuristic in line 23 diversifies the service composition. The two latter heuristics are unchanged from the versions in Brouer et al. [6].

3.1. The improvement heuristic with level of service considerations

The integer program described in line 8 of Algorithm 1 is a move operator in a large-scale neighborhood search based on altering a single service at a time. The objective of the integer program are estimation functions for changes in the flow of the network and the duration of the service due to insertions and removals of port calls. The solution of the integer program provides a set of moves in the composition of port calls and fleet deployment. Flow changes and the resulting change in the revenue for relevant commodities to the insertion/removal of a port call are estimated

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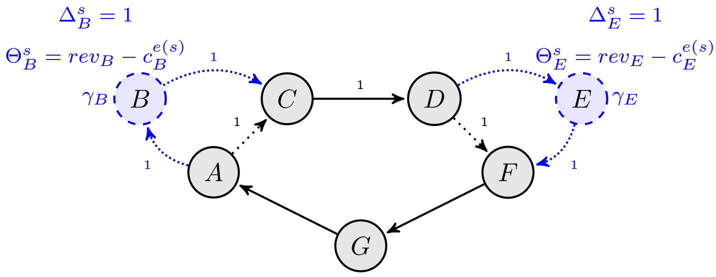
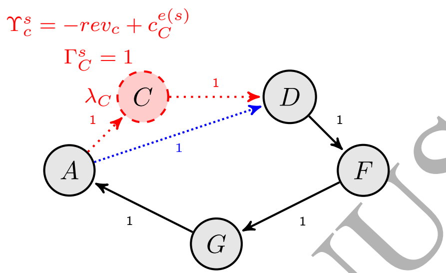


(a) Blue nodes (dashed) are evaluated for insertion corresponding to variables γ_i for the set of ports in the neighborhood N^s of service s.



(b) Red nodes (dashed) are evaluated for removal corresponding to variables λ_i for the set of current port calls F^s on service s.

Figure 2: Illustration of the estimation functions for insertion and removal of port calls.

by solving a series of resource constrained shortest path problems considering feasibility of transit time restrictions as well as the cost of transport including transshipments.

Given a total estimated change in revenue of rev_i and port call cost of $c_i^{e(s)}$ Figure 2(a) illustrates estimation functions for the change in revenue (Θ_i^s) and duration increase (Δ_i^s) for inserting port i into service s controlled by the binary variable γ_i. The duration controls the number of vessels needed to maintain a weekly frequency. Figure 2(b) illustrates the estimation functions for the change in revenue (Υ_i^s) and decrease in duration (Γ_i^s) for removing port i from service s controlled by the binary variable λ_i.

For considering the transit time in the IP, it is necessary to estimate how insertions and removals of port calls will affect the duration of the existing flow on the service. If an insertion is estimated to result in exceeding the transit time restriction of existing flow, and there is no possibility of rerouting the flow on a different path respecting the transit time limits, a loss of revenue can be expected. The loss is estimated to correspond to the full revenue obtained from the demand quantity. Figure 3 illustrates a case of a path variable in the current basis of the MCF model, which becomes infeasible due to transit time restrictions when inserting port B on its path.

In order to account for the transit time restrictions of the current flow, constraints (8) are added to the IP and a penalty, ζ_x corresponding to losing the cargo, is added to the objective if the transit time slack for an existing path variable becomes negative. This is handled through the variable α_x, where x refers to a path variable with positive flow in the current solution and s_x refers to the current slack time according to the transit time restrictions of the variable. Variable speed is considered in the estimation function for the flow as well as for the estimation of the service duration. The speed on the sailings to and from the port, i, evaluated for insertion is estimated to be equal to the speed sailed between the two ports previously connected and is denoted by the

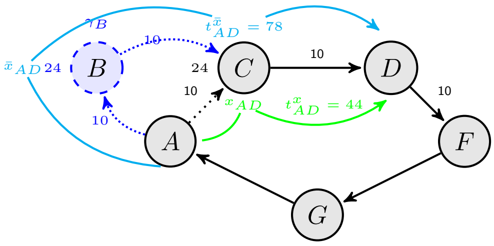
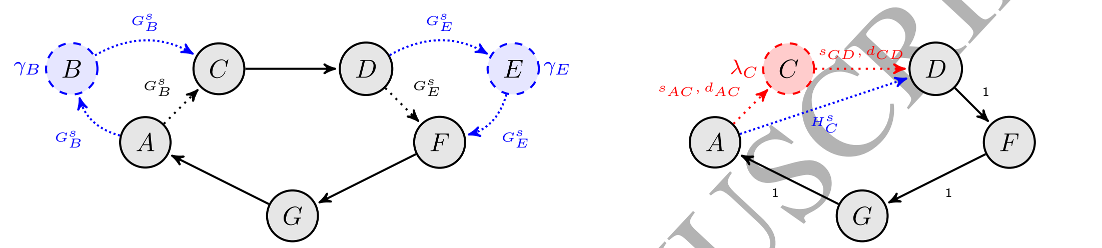


Figure 3: Insertions/removals affect transit time of the flow. Commodity k_{AD} has a maximum transit time $\bar{t}_k$ of 48 hours and the insertion of γ_B will make path variable x_{AD} infeasible.



(a) Blue nodes (dashed) are evaluated for insertion corresponding to variables γ_i for the set of ports in the neighborhood N^s of service s. Speeds of sailings to and from the insertion correspond to the speed of the existing link.

(b) Red nodes (dashed) are evaluated for removal corresponding to variables λ_i for the set of current port calls F^s on service s. A weighted average speed is used $H^s_C = (d_{AC}d_{CD})/(d_{AC} + d_{CD}) + (d_{CD}d_{DC})/(d_{AC} + d_{CD})$.

Figure 4: Illustration of the speeds used by estimation functions for insertion and removal of port calls.

parameter G^s_i. Upon evaluating a removal of a port the actual speed of the sailing in question is used to reduce the duration of the service. The parameter H^s_i expresses the weighted average speed of the current speeds for the sailings entering and leaving the port, i, estimated for removal. The speeds used for the estimation functions are illustrated in Figure 4.

For ease of reading, Table 1 gives an overview of additional sets, constants, and variables used in the CLSNDP model.

The objective of the move operator is to maximize the estimated profit increase obtained from removing and inserting port calls, accounting for the estimated change of revenue, transshipment cost, port call cost, and fleet cost.

$$\max \sum_{i \in N^s} \Theta_i \gamma_i + \sum_{i \in F^s} \Upsilon_i \lambda_i - C^{e(s)} \omega^s - \zeta_x \alpha_x \tag{1}$$

First, we need to estimate the number of vessels ω^s needed on the service s (assuming a weekly frequency) after insertions/removals while accounting for the change in the service time given the current weighted average speed on the service V^s:

$$\frac{Y^s}{V^s} + \sum_{i \in F^s} B_i + \sum_{i \in N^s} \left(\frac{\Delta^s_i}{G^s_i} + B_i \right) \gamma_i - \sum_{i \in F^s} \left(\frac{\Gamma^s_i}{H^s_i} + B_i \right) \lambda_i \leq 24 \cdot 7 \cdot (D^{e(s)} + \omega^s) \tag{2}$$

Next, we must ensure that the solution does not exceed the available fleet of vessels. Note that ω^s does not need to be bounded from below by $-D^{e(s)}$ because it is not allowed to remove all port calls:

$$\omega^s \leq M^{e(s)} \tag{3}$$

Then, a limit on the number of port call insertions and removals is enforced in order to minimize the error in the computed estimates:

$$\sum_{i \in N^s} \gamma_i \leq I^s \tag{4}$$

$$\sum_{i \in F^s} \lambda_i \leq R^s \tag{5}$$

Furthermore, the flow estimates are based on cargo flowing to and from a set of related port calls on the service. The affected ports are placed in a lock set, L_i, for insertions and removals respectively, i.e. ports in a lock set cannot be removed to avoid large deviations in the flow estimates:

$$\sum_{j \in L_i} \lambda_j \leq |L_i|(1 - \gamma_i) \qquad i \in N^s \tag{6}$$

$$\sum_{j \in L_i} \lambda_j \leq |L_i|(1 - \lambda_i) \qquad i \in F^s \tag{7}$$

Sets	
K	Set of all commodities k, each having origin o_k and destination d_k
S	Current set of services s, each service having vessel class $e(s)$
F^s	Set of port calls in service s
N^s	Set of neighbors (potential port call insertions) of service s
X^s	Set of path variables in current flow solution with positive flow associated with service s
$N^x \subseteq N^s$	Subset of neighbors with insertion on current path of variable $x \in X^s$
$F^x \subseteq F^s$	Subset of port calls on current path of variable $x \in X^s$
L_i	Lock set for port call insertion $i \in N^s$ or port call removal $i \in F^s$
Constants	
Y^s	Distance of the route associated with service s
B_i	Berthing time for port call $i \in F^s$
V^s	Estimated weighted average speed over all sailings on the service s
G_i^s	Speed on sailing to and from port i inserted on the service s
H_i^s	Speed on sailing to and from the port i removed from the service s
C^e	Cost of an additional vessel of class e
D^e	Number of deployed vessels of class e to a service in the current solution
M^e	Number of undeployed vessels of class e in the current solution
I^s	Maximum number of insertions allowed in s
R^s	Maximum number of removals allowed in s
Δ_i^s	Estimated distance increase if port call $i \in N^s$ is inserted in s
Γ_i^s	Estimated distance decrease if port call $i \in F^s$ is removed from s
Θ_i	Estimated profit increase of inserting port call $i \in N^s$ in s
Υ_i	Estimated profit increase of removing port call $i \in F^s$ from s
ζ_x	Estimated penalty for cargo lost due to transit time
s_x	Slack time of path variable x
$\bar{t}_k$	Upper bound on the transit time for commodity k
Variables	
λ_i	(binary) 1 if port call $i \in F^s$ is removed from s, 0 otherwise
γ_i	(binary) 1 if port call $i \in N^s$ is inserted in s, 0 otherwise
ω^s	(integer) number of vessels added (removed if negative) to s
α_x	(binary) 1 if transit time of path variable $x \in X^s$ is violated, 0 otherwise

Table 1: Overview of sets, constants, and variables used in the IP model for CLSNDP. Service s and vessel class e is written in superscript, while ports p and arcs (i, j) are written as subscript.

Finally, we need to activate the estimated penalty for lost cargo due to an estimated violation of the transit time for the commodity on this particular path:

$$\sum_{i \in N^x} \left(\frac{\Delta_i^s}{V^s} + B_i \right) \gamma_i - \sum_{i \in F^x} \left(\frac{\Gamma_i^s}{V^s} + B_i \right) \lambda_i - \bar{t}_k \alpha_x \leq s_x \qquad x \in X^s \tag{8}$$

The domains of the variables are:

$$\lambda_i \in \{0,1\},\ i \in F^s \quad \gamma_i \in \{0,1\},\ i \in N^s \quad \alpha_x \in \{0,1\},\ x \in X^s \quad \omega^s \in \mathbb{Z},\ s \in S \tag{9}$$

Thus the move operator is both guided by the changes in revenue due to new or removed connections as in Brouer et al. [6], but also by the change in revenue related to not transporting cargo for which the path duration is estimated to exceed the transit time of the commodity. All the sets in Table 1 are dynamic following the changes made to service s either by removal, where all $\lambda_i = 1$ will be removed from the set F_s. Similarly, insertions $\gamma_i = 1$ are added to the set of port calls F_s (and removed from the set of neighbors N^s) in the subsequent iteration, when a modified MIP is solved for the modified service s. The set of neighbours N^s will be defined according to the modified F_s and the set of current path variables X^s are updated to reflect the current commodity flow in each iteration. Likewise, the locksets L_i are depending on the estimations of changes to the commodity flow in the current iteration when calculating Θ and Υ. We refer to Karsten et al. [14] for a detailed explanation of these calculations.

3.2. Variable Speed on Service Legs

To include variable speed in the matheuristic (Algorithm 1 line 10) we formulate the speed optimization problem as a mixed integer program with a non-linear objective function that can easily be solved for each service $s \in S$ during the iterative search.

The MIP problem is solved independantly for each service s with corresponding vessel class e given by $e(s)$. For ease of reading we therefore remove the superscript s to all variables, except when refering to constants and variables from the original model (1)–(9) defined in Table 1.

Let m be the number of port calls in the round trip of s, and let $m+1$ correspond to the first port in the trip. The entry time t_{o_k} for a commodity on a service is smaller or larger than the exit time t_{d_k} depending on the starting port of a service. The function $g(t_{j,j+1}, d_{j,j+1})$ represents the bunker consumption from port j to $j+1$ expressed as a function of sailing time $t_{j,j+1}$ and distance $d_{j,j+1}$, which indirectly models the speed $v_{j,j+1}$. For each service we wish to determine the sailing speed of each sailing leg which we do by finding the optimal sailing time $t_{j,j+1}$ between ports j and $j+1$. We arrive in port j at time t_j and the sailing time must be determined such that the weekly frequency of a service is maintained. If the sailing speed is changed significantly it is possible to add or remove an additional vessel to the service provided that additional vessels are available. We also consider commodities that are not currently transported but could be transported on service s if a sufficient speed increase is profitable. To find the set of candidate commodities for a service we solve an unconstrained shortest path problem on the residual capacity graph of the current network for all commodities that are not currently transported. We add the ones that have a profitable path through service s to the set but where transit time is violated to $\bar{K}'$ and calculate the potential profit based on the residual capacity (which may be less than the demand of a cargo), the cost of the path and the service penalty (which we potentially can avoid). Additionally we keep track of the time decrease needed (corresponding to a speed up) to make the path feasible. The constants, sets and variables used in the model for a specific service $s \in S$ are summarized in Table 2.

Using this notation, the objective for each service s is to minimize the objective function accounting for the bunker cost, the expected loss of revenue due to transit times not met and

Sets	
K'	Set of commodities currently transported on service s where $t_{o_k} < t_{d_k}$
K''	Set of commodities currently transported on service s where $t_{o_k} > t_{d_k}$
$\bar{K}'$	Set of commodities that potentially could be transported on s where $t_{o_k} < t_{d_k}$
$\bar{K}''$	Set of commodities that potentially could be transported on s where $t_{o_k} > t_{d_k}$
Constants	
m	Number of port calls in round trip of service s
$d_{j,j+1}$	Distance between ports j and $j+1$ using vessel class e
t_{max}	Maximum time to complete service s sailing at minimum speed
t_k	Time commodity k currently uses on service s and the possible slack time between the time of the current path and the overall transit time limit of k
β	The price for one metric ton of bunker
z_k	Net revenue that will be lost if not transporting the demand $k \in K' \cup K''$
r_k	Net revenue that can be obtained by transporting all of demand $k \in \bar{K}' \cup \bar{K}''$
τ_k	Time commodity $k \in \bar{K}' \cup \bar{K}''$ currently would spend on service s
τ'_k	Time currently lacking for commodity $k \in \bar{K}' \cup \bar{K}''$
v_{max}	Maximum vessel speed for vessel class e
v_{min}	Minimum vessel speed for vessel class e
Variables	
t'_j	(continuous) Arrival time at port j for service s
$t_{j,j+1}$	(continuous) Sailing time between ports j and $j+1$ for service s
δ	(integer) Change in the number of vessels of class e deployed to service s
ρ_k	(binary) 1 if commodity k will be lost due to transit time violation
η_k	(binary) 1 if commodity k will be available if transit time is reduced

Table 2: Overview of sets, constants, and variables used in the speed MIP. All definitions refer to a given service s and corresponding vessel class e.

the deployment cost of additional vessels less the profit from demand that become available for transport by adjusting the speed. The objective can be written as:

$$\min \quad \beta \sum_{j=1}^{m} g^e(t_{j,j+1}, d_{j,j+1}) + \sum_{k \in K' \cup K''} z_k \rho_k + C^e \delta - \sum_{k \in \bar{K}' \cup \bar{K}''} r_k \eta_k \tag{10}$$

A number of constraints need to be satisfied: First, we need to set the time for each port on a route and the sailing time between ports for calculating the bunker consumption:

$$t'_{j+1} - t'_j - t_{j,j+1} = B_j \qquad j = 1 \dots m \tag{11}$$

Next, we decide the number of vessels needed to maintain a weekly frequency on the service including berthing time for each port call:

$$t'_{m+1} - 24 \cdot 7 \cdot \delta = 24 \cdot 7 \cdot D^e - \sum_{j=1}^{m} B_j \tag{12}$$

The service time is set by the constraint:

$$\sum_{j=1}^{m} t_{j,j+1} = t'_{m+1} \tag{13}$$

Moreover, we invoke a loss of revenue if the transit times of commodities on board the service s are not met. A separate constraint is necessary for commodities where $t'_{o_k} < t'_{d_k}$ to account for the total round trip time:

$$t'_{d_k} - t'_{o_k} - \rho_k t_{max} \leq t'_k \qquad k \in K' \tag{14}$$

$$t'_{d_k} - t'_{o_k} - \rho_k t_{max} + t'_{m+1} \leq t'_k \qquad k \in K'' \tag{15}$$

Similar constraints allow a service to pick-up additional cargo if speed is increased sufficiently to make paths for cargo that was previously rejected due to transit time limits:

$$t'_{d_k} - t'_{o_k} - (1 - \eta_k) t_{max} \leq \tau_k - \tau'_k \qquad k \in \bar{K}' \tag{16}$$

$$t'_{d_k} - t'_{o_k} - (1 - \eta_k) t_{max} + t'_{m+1} \leq \tau_k - \tau'_k \qquad k \in \bar{K}'' \tag{17}$$

Finally, we need to enforce speed bounds of the vessel class used by service s:

$$t_{j,j+1} \geq \frac{d_{j,j+1}}{v_{max}} \qquad j = 1 \dots m \tag{18}$$

$$t_{j,j+1} \leq \frac{d_{j,j+1}}{v_{min}} \qquad j = 1 \dots m \tag{19}$$

The variable δ is bounded from above by the number of available vessels if the service slows down overall by adding an additional vessel to the service. The bounds on δ are tightened in order to give a good solution close to the current deployment such that $-1 \leq \delta \leq min\{1, M^e\}$, i.e. it is only possible to add or remove at most one vessel. The variable domains are:

$$\delta \in \{-1, 0, \min\{1, M^e\}\} \tag{20}$$

$$t'_j, t_{j,j+1} \in \mathbb{R}^+ \qquad j = 1 \dots m \tag{21}$$

$$\rho_k \in \{0, 1\} \qquad k \in K' \cup K'' \tag{22}$$

$$\eta_k \in \{0, 1\} \qquad k \in \bar{K}' \cup \bar{K}'' \tag{23}$$

Category	Instance and description	$\|P\|$	$\|K\|$	$\|E\|$
Single-hub	**Baltic** Baltic sea, Bremerhaven as hub	12	22	2
	WAF West Africa, Algeciras as hub	19	38	2
Multi-hub	**Mediterranean** Algeciras, Tangier, and Gioia Tauro as hubs	39	369	3
Trade-lane	**Pacific** Asia and US West Coast	45	722	4
	AsiaEurope Europe, Middle East and Far East regions	111	4000	6
World	**WorldSmall** 47 main ports worldwide	47	1764	6

Table 3: The instances of the benchmark suite with indication of the number of ports $|P|$, the number of origin-destination pairs $|K|$, and the number of vessel classes $|E|$.

The objective function can be linearized by modeling the bunker consumption as a piecewise linear function for each $t_{j,j+1}$ and the model (10)–(23) can be solved efficiently by a standard mixed integer programming solver. We use 100 pieces to accurately model the bunker consumption function (the solution times for the speed optimization problem are generally less than 0.1 seconds in the instances we have solved in Section 4 and the number of pieces used to aprroximate the objective only has limited impact on this.)

As described earlier, when a service in the network is changed we re-solve the cargo flowing subproblem using a warmstarting procedure where previously generated columns are used leading to a very effective solution of the flow problem. It should be noted that solving the speed optimization for each service separately leads to a sub-optimal configuration of the network as a significant portion of the demands uses more than one service and hence the transit time for each demand is determined by more than one service, but as we solve the problem many times for each service as part of the search procedure large differences can be reduced.

4. Computational Results

The matheuristic was tested on data from the benchmark suite *LINER-LIB* described in Brouer et al. [4]. The instances can be found at *http://www.linerlib.org*. Table 3 gives an overview of the instances. The transit time restrictions have been updated according to the most recent published liner shipping transit times for a small number of the origin-destination pairs as described in Karsten et al. [14].

The matheuristic has been coded in `C++` and run on a linux system with an *Intel(R) Xeon(R) X5550* CPU at *2.67GHz* and *24 GB* RAM. The algorithm is set to terminate after the time limits imposed in Brouer et al. [4] if the stopping criterion of the embedded simulated annealing procedure is not fulfilled at the time limit.

We fix the berthing time, B_p to 24 hours for all ports as in Brouer et al. [4] and the transshipment time, t_a is fixed to 48 hours for every connection as the concrete time schedule is not known at this stage. The bunker price is set to \$ 600 per ton as in Brouer et al. [4]. Prices for bunker have nearly halved in the past five years, and to this end Section 4.2 is a case study of key performance indicators for networks constructed with bunker prices ranging from \$ 150 to \$ 700 per ton.

4.1. *Computational results for* LINER-LIB

Table 4 shows the performance of the algorithm on the six instances described in Table 3. For each instance the performance of the algorithm is shown when the networks are designed with constant and variable speed. We evaluate the average performance of ten networks in the two settings and also report the best found network. In both the constant speed and variable speed setting the algorithm can find profitable solutions (negative objective values) for Baltic, WAF, WorldSmall, and AsiaEurope. The Pacific instance yields unprofitable solutions though both fleet deployment and transported cargo volume is high. For all instances except the single-hub instances the networks generated with variable speed are consistently better than the constant speed network with an improvement of up to 10% for the average values and up to a more than 60 % better objective value for the best Pacific network. On average around 85% to 95% of the available cargo volume is transported except in the Mediterranean instance. Generally the constant speed instances transport slightly more of the cargo volume than the networks operating at variable speed and the fleet deployment is significantly higher for networks operating at variable speed suggesting overall slower sailing speed. This is also evident from Table 5 where the weighted average speed for each vessel class is shown for networks with constant and variable speed. Most of the vessel classes sail significantly slower for the larger networks and variable speed networks generally operate around or below design speed whereas the networks with constant speed operate at or in some cases much above design speed.

Table 6 gives statistics on the rejected cargo in the networks with variable speed. The reasons for cargo to be rejected is that there are no cargo paths that meet transit time restrictions, that there is no residual capacity or that the origin-destination pair is not connected in the graph. For Baltic, WAF, and Mediterranean cargo is primarily rejected because the corresponding origin-destination pairs are not connected. This indicates that there is a set of ports that the algorithm asses to be unprofitable to call. For Pacific, WorldSmall, and AsiaEurope cargo is mainly not transported because of transit times that cannot be met but also to a large degree because of lacking capacity. For these only around 25 % is rejected because of no connections. Generally for the cargo that is rejected because of no connection the percentage of rejected demands in terms of number of demands (k) compared to the volume (v) not connected show that there is a lot of low volume cargo here. Further inspection shows that these demands often are from smaller feeder ports where the total available volume is very low which is why they are assessed to be unprofitable by the algorithm.

4.2. Sensitivity to Bunker Price

The price of bunker is very decisive for the cost of the network and the soaring oil prices of more than $ 600 per ton seen at the beginning of this decade along with a surplus of capacity in the market gave rise to the "slow-steaming" era. Recently, oil prices have been plummeting to less than $ 300 per ton, which means that the trade-off between slow steaming by deploying extra vessels and speeding up services is shifting. This section concerns the performance of the algorithm with a varying price of bunker. The test is performed on several WorldSmall instances, where we are using the same initial solutions for different bunker prices. The subsequent improvement heuristic will be highly dependent on the bunker price in evaluating a given move and the best found solutions will potentially differ significantly. We compare solutions for bunker prices in the range from $ 150 to $ 700 per ton in terms of vessel deployment, the percentage of cargo transported, and the weighted average speed of the network.

Table 7 and Figure 5 show the correlation between bunker price and the profit margin, which is decreasing with increasing bunker prices. Furthermore, it can be seen that the amount of available cargo transported only decrease a few percent with more then a quadrupling of the bunker price. Table 8 gives statistics on why cargo is rejected, and as expected more cargo is rejected due to transit times at higher bunker cost, because it is more favourable to slow down than meeting tight connections.

In Table 9 and Figure 6 the expected trend of a decreasing speed with an increasing bunker price is clearly seen for all vessel classes except the SuperP class. The weighted average speed confirms this trend when vessel time charter cost is kept constant. When bunker price is low, average speed increase, meaning that the overall deployment also can decrease as fewer vessels are needed.

Instance	**Obj. Val.**	**Deployment**		**Transp. Vol.**	**CPU Time**
	Z(7)	**D(v)** (%)	**D(|*E*|)** (%)	**T(v)** (%)	**(S)**
		Baltic			
Best (constant speed)	$-1.41 \cdot 10^4$	100	100	87.4	101
Average (constant speed)	$7.45 \cdot 10^4$	100	100	86.7	108
Best (variable speed)	$-0.46 \cdot 10^4$	100	100	87.9	144
Average (variable speed)	$17.4 \cdot 10^4$	100	100	85.1	115
		WAF			
Best (constant speed)	$-5.59 \cdot 10^6$	83.3	85.7	97.0	255
Average (constant speed)	$-4.87 \cdot 10^6$	83.3	85.2	94.3	354
Best (variable speed)	$-5.48 \cdot 10^6$	97.2	97.6	97.6	362
Average (variable speed)	$-4.89 \cdot 10^6$	86.2	87.6	91.7	396
		Mediterranean			
Best (constant speed)	$2.42 \cdot 10^6$	91.9	95.0	86.9	710
Average (constant speed)	$2.70 \cdot 10^6$	90.5	94.0	78.9	737
Best (variable speed)	$2.19 \cdot 10^6$	91.9	95.0	83.8	1200
Average (variable speed)	$2.65 \cdot 10^6$	92.5	95.0	79.8	1200
		Pacific			
Best (constant speed)	$3.05 \cdot 10^6$	95.0	91.0	93.3	3600
Average (constant speed)	$3.65 \cdot 10^6$	94.0	91.9	94.0	3600
Best (variable speed)	$1.13 \cdot 10^6$	98.2	97.0	90.3	3600
Average (variable speed)	$3.44 \cdot 10^6$	97.0	96.0	89.5	3600
		WorldSmall			
Best (constant speed)	$-3.54 \cdot 10^7$	82.0	85.2	91.1	10800
Average (constant speed)	$-3.15 \cdot 10^7$	82.3	85.4	90.9	10800
Best (variable speed)	$-4.05 \cdot 10^7$	90.5	96.6	89.1	10800
Average (variable speed)	$-3.48 \cdot 10^7$	90.3	95.8	88.0	10800
		AsiaEurope			
Best (constant speed)	$-1.67 \cdot 10^7$	84.6	90.9	88.8	14400
Average (constant speed)	$-1.45 \cdot 10^7$	83.9	91.9	88.5	14400
Best (variable speed)	$-1.88 \cdot 10^7$	94.4	96.0	85.6	14400
Average (variable speed)	$-1.52 \cdot 10^7$	94.0	96.8	84.9	14400

Table 4: Best and average of 10 runs on an *Intel(R) Xeon(R) X5550* CPU at *2.67GHz* with *24 GB* RAM. Results with constant and variable speed. Weekly objective value (**Z(7)**); percentage of fleet deployed as a percentage of the total volume **D(v)** and as a percentage of the number of ships **D(|*E*|)**. **T(v)** is the percentage of total cargo volume transported and (**S**) is the execution time in CPU seconds.

This is also seen in Figure 6. The algorithm performs as expected under varying conditions and confirms that even under very different economics conditions we can design profitable networks. The characteristics in terms of deployment and sailing speed of these networks is rather different, but in all cases the algorithm is able to design networks with a high transportation percentage. It should be noted that in these tests only the bunker price is varied while in a real setting the freight rates also depend on the bunker price leading to different network characteristics. However, the sensitivity analysis illustrates how the algorithm also can be used as a managerial tool to conduct "what if" analyses at a strategic level.

Instance	Vessel Class					
	F450	**F800**	**P1200**	**P2400**	**PostP**	**SuperP**
Baltic						
Constant Speed	10.8	13.7				
Variable Speed	11.1	13.9				
WAF						
Constant Speed	11.5	13.2				
Variable Speed	10.8	11.7				
Mediterranean						
Constant Speed	11.9	13.7	13.9			
Variable Speed	11.7	13.0	15.5			
Pacific						
Constant Speed	12.0	14.2	15.9	18.2		
Variable Speed	11.2	12.4	14.9	15.6		
WorldSmall						
Constant Speed	12.7	15.5	17.5	19.4	19.4	18.2
Variable Speed	12.0	13.2	16.4	16.4	15.8	15.6
AsiaEurope						
Constant Speed	11.7	13.7	16.5	18.0	19.7	17.6
Variable Speed	11.5	12.8	16.1	14.8	16.6	15.8
Class Characteristics						
Design Speed	**12.0**	**14.0**	**18.0**	**16.0**	**16.5**	**17.0**
Max speed	**14.0**	**17.0**	**19.0**	**22.0**	**23.0**	**22.0**

Table 5: Weighted average speed per vessel class over ten runs. The last two rows indicate the design speed and max speed of the corresponding vessel class. F is Feeder, P is Panamax.

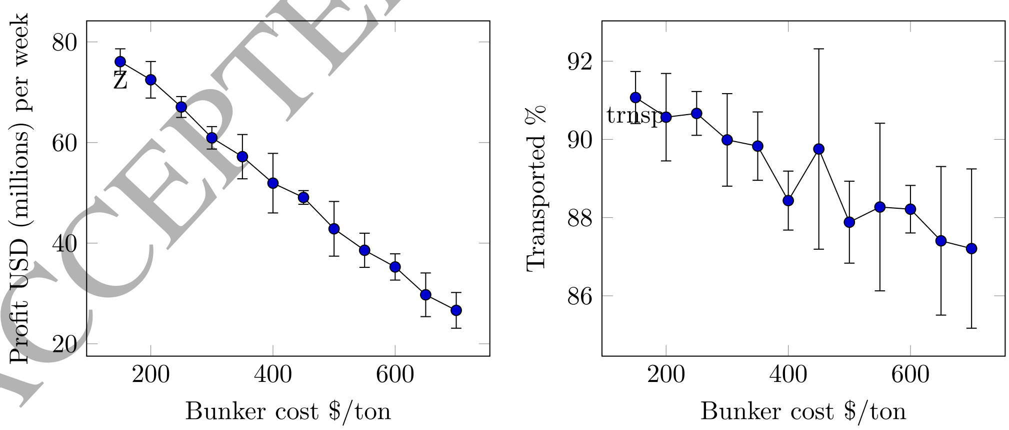


Figure 5: (left) Development in objective value **Z**, and (right) cargo transported in percentage of total available **trnsp**, as function of bunker cost. The results are average of five runs (error bars represent the standard deviation).

Instance		Total rejected		Transit time		Capacity		Transit time and capacity		Not connected	
		|**R**|	**FFE**	**tt(k)** (%)	**tt(v)** (%)	**C(k)** (%)	**C(v)** (%)	**ttC(k)** (%)	**ttC(v)** (%)	**L(k)** (%)	**L(v)** (%)
Baltic	μ	8	732	1.1	0.2	22.6	77.1	0.0	0.0	76.3	22.7
	σ	1	164	3.5	0.6	11.5	10.4	0.0	0.0	14.1	10.6
WAF	μ	8	712	7.0	1.2	14.0	26.1	1.7	0.1	77.3	72.6
	σ	2	314	12.1	2.2	9.7	25.3	5.3	0.3	13.5	24.9
Mediterranean	μ	107	1527	35.3	50.0	0.2	0.4	4.3	4.0	60.1	45.7
	σ	8	250	7.2	9.6	0.7	1.0	4.6	3.9	5.9	8.6
Pacific	μ	240	4657	51.5	34.4	7.8	27.4	13.3	29.9	27.3	8.3
	σ	23	641	6.7	7.4	3.3	12.6	4.1	11.8	5.9	3.4
WorldSmall	μ	325	15334	35.8	40.2	19.9	16.7	21.1	23.9	23.2	19.2
	σ	45	1872	6.5	8.3	9.4	8.3	11.1	11.7	20.4	43.5
EuropeAsia	μ	1029	11597	41.9	44.9	8.4	14.3	21.3	26.4	28.4	14.4
	σ	97	1008	7.5	8.3	2.9	3.5	5.8	7.3	8.5	6.3

Table 6: Statistics on the rejected demand reporting average (μ) and standard deviation (σ) over ten runs. $|\mathbf{R}|$ is the number of rejected OD pairs and **FFE** is the corresponding rejected volume; **tt(k)** is the percentage of OD pairs rejected due only to transit time and **tt(v)** is the corresponding percentage of the total volume; **C(k)** is the percentage of OD pairs rejected due only to lack of capacity and **C(v)** is corresponding percentage of the total volume; **ttC(k)** is the percentage of OD pairs rejected due to both transit time and lack of capacity and **ttC(v)** is the corresponding percentage of the total volume; **L(k)** is the percentage of OD pairs not connected and **L(v)** is the corresponding percentage of the total volume.

Bunker	**Obj. Val.**	**Deployment**		**Transp. Vol.**
Price ($/ton)	**Z(7)** ($)	**D(v)** (%)	**D($\|E\|$)** (%)	**T(v)** (%)
150	$7.67 \cdot 10^7$	91.8	95.6	90.3
200	$7.24 \cdot 10^7$	90.2	95.1	90.1
250	$6.85 \cdot 10^7$	91.0	95.3	89.8
300	$6.45 \cdot 10^7$	93.5	96.3	91.1
350	$5.81 \cdot 10^7$	94.4	95.9	89.9
400	$5.20 \cdot 10^7$	91.3	96.3	88.9
450	$4.86 \cdot 10^7$	95.0	97.3	89.3
500	$4.39 \cdot 10^7$	95.0	97.4	88.7
550	$4.15 \cdot 10^7$	94.8	96.9	89.3
600	$3.54 \cdot 10^7$	93.0	96.0	88.4
650	$2.90 \cdot 10^7$	91.5	96.2	86.2
700	$2.26 \cdot 10^7$	93.7	96.7	85.7

Table 7: Bunker price and the development in the objective value **Z(7)**, deployment percentage of volume **D(v)** and number of vessels **D($|E|$)** and the percentage of cargo transported **T(v)**. Average of five different runs.

The red trend lines in Figure 6 show linear fits of the speed ($f(x) = -0.002x+16.8$), deployment ($f(x) = 0.002x + 95.2$), and amount of transported cargo ($f(x) = -0.008x + 92.2$). These linear approximations confirm the expectation that speed decrease with increased bunker price (0.2 nm/h per 100 $/ton increase), the amount transported decrease with increased bunker price (0.8 % per 100 $/ton increase), and deployment increase with increased bunker price (0.2 % per 100 $/ton increase). This is expected as the bunker consumption per unit of distance is a quadratic function of speed. When the bunker price increases, we need more vessels as the network tend to operate at lower speeds. This also implies that some demands cannot meet their transit times even with different service layouts.

Bunker price	Total rejected		Transit time		Capacity		Transit time and capacity		Not connected	
	$\lvert\mathbf{R}\rvert$	**FFE**	**tt(k)** (%)	**tt(v)** (%)	**C(k)** (%)	**C(v)** (%)	**ttC(k)** (%)	**ttC(v)** (%)	**L(k)** (%)	**L(v)** (%)
150	280	12443	33,0	37,4	21,9	21,1	16,5	19,3	28,6	22,2
200	264	12638	41,2	48,3	31,3	24,3	17,7	21,1	9,8	6,4
250	281	13025	38,6	45,0	22,9	17,9	18,3	22,3	20,3	14,8
300	254	11408	43,5	49,4	25,4	21,2	21,9	23,3	9,2	6,1
350	277	12963	47,1	48,1	27,7	21,1	20,7	27,7	4,5	2,9
400	305	14228	49,4	55,6	15,4	11,6	13,5	17,6	21,8	15,3
450	295	13776	38,6	41,2	22,5	17,0	21,7	30,0	17,2	11,8
500	303	14523	50,0	52,4	21,7	18,2	17,3	21,9	10,9	7,3
550	299	13720	43,5	44,0	24,7	20,9	27,8	31,8	3,9	3,1
600	319	14902	40,2	42,0	15,5	15,5	21,3	26,6	23,0	15,9
650	374	17709	53,3	61,0	18,7	13,3	16,3	17,9	11,7	7,9
700	382	18310	46,0	50,3	18,2	18,3	18,6	20,7	17,3	10,7

Table 8: Rejected demand given the difference in bunker price. $\lvert\mathbf{R}\rvert$ is the number of rejected OD pairs and **FFE** is the corresponding rejected volume; **tt(k)** is the percentage of OD pairs rejected due only to transit time and **tt(v)** is the corresponding percentage of the total volume; **C(k)** is the percentage of OD pairs rejected due only to lack of capacity and **C(v)** is corresponding percentage of the total volume; **ttC(k)** is the percentage of OD pairs rejected due to both transit time and lack of capacity and **ttC(v)** is the corresponding percentage of the total volume; **L(k)** is the percentage of OD pairs not connected and **L(v)** is the corresponding percentage of the total volume. The results are an average of five runs.

$/ton	F450	#v	F800	#v	P1200	#v	P2400	#v	PostP	#v	SuperP	#v	Total V	W. Av. S.
150	11,8	24	14,0	29	17,2	66	17,8	74	17,3	53	16,8	7	251	16,5
200	11,9	24	13,5	29	17,1	67	17,8	74	17,5	50	14,0	7	250	16,5
250	11,8	24	13,3	29	16,7	67	17,2	72	16,9	53	13,0	6	251	16,0
300	11,9	24	13,2	28	16,6	65	17,6	74	16,8	55	18,6	7	253	16,2
350	12,2	24	13,3	29	16,4	64	16,6	73	16,6	53	16,2	9	252	15,7
400	11,5	24	13,7	29	16,4	68	16,7	73	16,4	55	12,4	5	253	15,7
450	11,5	24	13,1	29	16,4	67	16,7	74	15,8	54	16,1	9	256	15,5
500	11,7	24	13,4	29	16,2	67	16,3	74	16,1	54	15,9	8	256	15,5
550	11,6	23	12,8	29	16,5	67	16,3	73	15,9	55	17,0	8	255	15,5
600	11,4	24	13,4	29	16,3	67	16,5	73	15,8	52	15,3	8	252	15,4
650	12,0	24	13,2	29	16,1	68	15,8	73	15,7	54	15,6	6	253	15,2
700	11,7	24	13,8	29	16,1	66	15,9	74	15,3	55	15,5	7	254	15,2

Table 9: Relation between bunker price, weighted average speed per vessel class and vessel deployment for each class. Weighted Average speed (W. Av. S.) is a weighted by the number of vessels deployed in the class (#v). The results are an average of five runs.

The sensitivity analysis illustrates how the incentives towards slow steaming for liner shipping companies change with varying bunker prices. It will be a more active choice to maintain a greener profile in periods with low oil prices as attaining *"an acceptable environmental performance in the transportation supply chain, while at the same time respecting traditional economic performance criteria"* (Psaraftis [22]) is only a win-win solution when oil prices are high.

5. Conclusion

We have presented the competitive liner shipping network design problem where we include level of service requirements in the form of tight transit time restrictions on all demands while maintaining the ability to transship between services. To improve the networks, getting more realistic transit times and a better fleet utilization, we propose a method that can handle variable

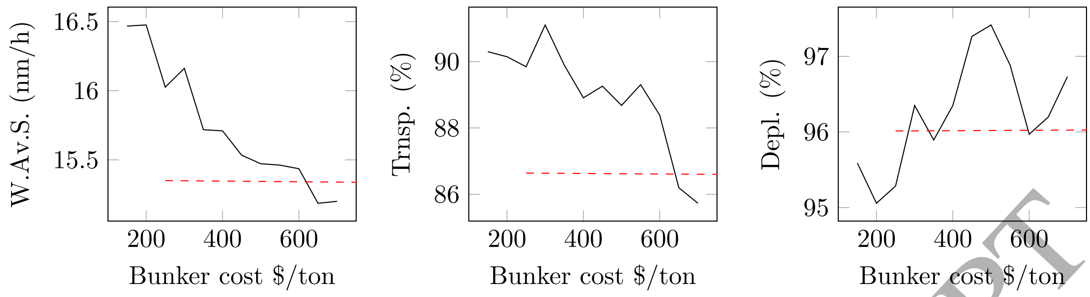


Figure 6: The weighted average speed (W.Av.S.), of an instance, the cargo transported in percentage of total available (Trnsp.), and the fleet capacity deployed in percentage of total volume, (Depl.) as a function of bunker price. The red dashed trend lines are based on a linear regression fit. The results are an average of five runs.

speed on all sailing legs in the network.

The proposed matheuristic can handle tight transit time restrictions on all demands and adjust speed on all sailing legs. The core components of the matheuristic is an integer program considering a set of removals and insertions to a service and an integer program that adjust the speed of each service iteratively. We extend the integer program to consider how removals and insertions influence the transit time of the existing cargo flow on the service. Each iteration of the matheuristic provides a set of moves for the current set of services and fleet deployment along with a proposed sailing speed on each service leg, which lead to a potential improvement in the overall profit. The evaluation of the cargo flow for a set of moves requires solving a time constrained multi-commodity flow problem using column generation.

Extensive computational tests, including a sensitivity analysis on bunker price, show that the algorithm is applicable in practice and that it is possible to generate profitable networks for the majority of the instances in *LINER-LIB* while considering level of service requirements. Especially for the larger instances the approach generates networks of good quality where the fleet is well utilized and the majority of demands are transported while satisfying transit time restrictions. Still, some smaller demands are not served and the fleet is not utilized completely, suggesting that further algorithmic improvements may lead to even better solutions. We expect that especially more flexibility in terms of possible vessel class swaps could improve the algorithmic performance and the quality of the generated networks.

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Appendix A. Mathematical model

In the following we introduce a mathematical formulation of the CLSNDP. This is partly based on Brouer et al. [5], Karsten et al. [14] and extends the problem description of the LSNDP presented in Brouer et al. [4] to handle transit times and variable speed. The model enforces a weekly frequency resulting in a weekly planning horizon.

The mathematical model of the CLSNPD relies on a set of service variables and a path flow formulation of the underlying time constrained multi-commodity flow problem as described in Karsten et al. [15].

We define a directed graph, $G(V, A)$, with vertices V corresponding to ports and arcs A. The set of arcs in the graph can be divided into (un)load arcs, transshipment arcs, sailing arcs, and forfeited arcs to reject demand. We associate with each arc $(i, j) \in A$ a cost c_{ij}, traversal time t_{ij}, and capacity C_{ij}. The arcs used by service s is denoted A^s.

Let Ω_k be the set of all feasible paths for commodity $k \in K$ including forfeiting the cargo. Let $\Omega(i, j)$ be the set of all paths using arc $(i, j) \in A$. The cost of a path ρ is denoted as c_ρ and it includes the revenue obtained by transporting one unit of commodity k sent along path $\rho \in \Omega_k$. The real variable x_ρ denotes the amount of commodity k sent along the path and the demand of commodity $k \in K$ is q_k. The weekly cost of a service is

$$c^s = D^{e(s)}C^{e(s)} + \sum_{(i,j)\in A^s} \left(\beta \cdot (h^{e(s)}B_j + g^{e(s)}(t_{ij}, d_{ij})) + c_j^{e(s)} + l_{ij}^{e(s)}\right)$$

accounting for fixed cost of deploying the vessels and the variable cost in terms of the bunker and port call cost of one round trip. Define binary service variables y^s indicating the inclusion of service $s \in S$ in the solution.

Then the mathematical model of the CLSNDP can be formulated as follows.

$$\min \quad \sum_{s\in S} c^s y^s + \sum_{k\in\mathbf{K}} \sum_{\rho\in\Omega_k} c_\rho x_\rho \tag{A.1}$$

$$\text{s.t.} \quad \sum_{\rho\in\Omega_k} x_\rho = q_k \qquad k \in K \tag{A.2}$$

$$\sum_{\rho\in\Omega(i,j)} x_\rho \leq U^{e(s)} y^s \qquad s \in S, (i, j) \in A^s \tag{A.3}$$

$$\sum_{s\in S:e(s)=e} D^{e(s)} y^s \leq \Psi^e \qquad e \in E \tag{A.4}$$

$$x_\rho \in \mathbb{R}^+ \qquad \rho \in \Omega_k,\ k \in K \tag{A.5}$$

$$y^s \in \{0, 1\} \qquad s \in S \tag{A.6}$$

The objective (A.1) minimizes cumulative service and cargo transportation cost. As the cargo transportation cost includes the revenue of transporting the cargo, this is equivalent to maximizing profit. *The cargo flow constraints* (A.2) along with non-negativity constraints (A.5) ensure that all cargo is either transported or forfeited. *The capacity constraints* (A.3) link the cargo paths with the service capacity installed in the transportation network. *The fleet availability constraints* (A.4) ensure that the selected services can be operated by the available fleet. Finally, constraints (A.5) and (A.6) define the variable domains.

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